

# Cosmic Censorship: The Role of Quantum Gravity

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## Abstract

The cosmic censorship hypothesis introduced by Penrose thirty years ago is still one of the most important open questions in *classical* general relativity. In this essay we put forward the idea that cosmic censorship is intrinsically a *quantum gravity* phenomena. To that end we construct a gedanken experiment in which cosmic censorship is violated within the purely *classical* framework of general relativity. We prove, however, that *quantum* effects restore the validity of the conjecture. This suggests that classical general relativity is inconsistent and that cosmic censorship might be enforced only by a quantum theory of gravity.

Spacetime singularities that arise in gravitational collapse are always hidden inside of black holes. This is the essence of the (weak) cosmic censorship conjecture, put forward by Penrose thirty years ago [1]. The conjecture, which is widely believed to be true, has become one of the corner stones of general relativity. Moreover, it is being envisaged as a basic principle of nature. However, despite the flurry of activity over the years, the validity of this conjecture is still an open question (see e.g., [2,3] for reviews).

The destruction of a black hole's event horizon is ruled out by this principle because it would expose the inner singularities to distant observers. Moreover, the horizon area of a black hole,  $A$ , is associated with an entropy  $S_{BH} = A/4\hbar$  (we use  $G = c = 1$ ). Therefore, without any obvious physical mechanism to compensate for the loss of the black-hole enormous entropy, the destruction of the black-hole event horizon would violate the

generalized second law (GSL) of thermodynamics [4]. For these two reasons, any process which seems, at first sight, to remove the black-hole horizon is expected to be unphysical. For the advocates of the cosmic censorship principle the task remains to find out how such candidate processes eventually fail to remove the horizon.

The main goal of this essay is to put forward the idea that the stability of the black-hole horizon, and the *cosmic censorship* principle are intrinsically *quantum* phenomenon. To that end, we construct a gedanken experiment in which cosmic censorship is being violated within the purely *classical* framework of general relativity. We prove, however, that *quantum* effects save the cosmic censorship principle.

One of the earliest attempts to eliminate the horizon of a black hole is due to Wald [5]. As is well-known, the Reissner-Nordström metric with  $M < Q$  (where  $M$  and  $Q$  are the mass and charge) does not contain an event horizon, and it therefore describes a naked singularity. Wald tried to “over-charge” an extremal black hole (characterized by  $Q = M$ ) by dropping into it a charged test particle whose charge-to-mass ratio is larger than unity. Wald considered the specific case of a particle which starts falling from spatial *infinity* (thus, the particle’s energy-at-infinity is larger than its rest mass). He has shown that this attempt to “over-charge” the black hole would fail because of the Coulomb potential barrier surrounding the black hole.

A more ‘dangerous’ version of Wald’s original gedanken experiment is one in which the charged particle is *slowly* lowered towards the black hole. In this case, the energy delivered to the black hole (the part contributed by the body’s rest mass, see below) can be *red-shifted* by letting the assimilation point approach the black-hole horizon. On the other hand, the particle’s charge is not redshifted by the gravitational field of the black hole. At a first sight the particle [with arbitrarily small (redshifted) mass-energy] is not hindered from entering the black hole and removing its horizon, thereby violating cosmic censorship.

Consider a charged body of rest mass  $\mu$ , charge  $q$ , and proper radius  $b$ , which is *slowly* descent into a (near extremal) black hole. The total energy  $\mathcal{E}$  (energy-at-infinity) of the body in a black-hole spacetime is made up of three contributions: 1)  $\mathcal{E}_0 = \mu(g_{00})^{1/2}$ , the energy

associated with the body's mass (red-shifted by the gravitational field); 2)  $\mathcal{E}_{elec} = eQ/r$ , the electrostatic interaction of the charged body with the external electric field; and 3)  $\mathcal{E}_{self}$ , the gravitationally induced self-energy of the charged body.

The physical origin of the third contribution,  $\mathcal{E}_{self}$ , is the distortion of the charge's long-range Coulomb field by the spacetime *curvature*. This results in a repulsive (i.e., directed away from the black hole) self-force in the black-hole background. A variety of techniques have been used to demonstrate this effect in black-hole spacetimes. In particular, the contribution of this effect to the particle's (self) energy in the Reissner-Nordström background is  $\mathcal{E}_{self} = Mq^2/2r^2$  [6].

The total energy of a charged particle at a proper distance  $\ell$  ( $\ell \ll r_+$ ) above the horizon is given by:

$$\mathcal{E}(\ell) = \frac{\mu\ell(r_+ - r_-)}{2r_+^2} + \frac{qQ}{r_+} - \frac{qQ\ell^2(r_+ - r_-)}{4r_+^4} + \frac{Mq^2}{2r_+^2}, \quad (1)$$

where  $r_{\pm}$  are the locations of the black-hole (event and inner) horizons. This expression is actually the effective potential governing the motion of a charged body in the black-hole spacetime. Provided  $qQ > 0$ , it has a *maximum* located at  $\ell = \ell^*(\mu, q; M, Q) = \mu r_+^2/qQ$ .

The most challenging situation for the cosmic censorship conjecture occurs when the charge-to-energy ratio of the captured particle is as large as possible. This can be achieved if one *slowly* lowers the body as close to the horizon as possible. However, an object suspended in the vicinity of a black hole is actually accelerated by virtue of its being prevented from falling freely along a geodesic. As first pointed out by Unruh and Wald [7], the object would feel isotropic thermal radiation, the well-known Unruh radiance [7]. As a consequence, buoyancy in the radiative black-hole environs will *prevent* lowering the object *slowly* all the way down to the horizon. It will float at a proper height  $\ell = b$ , almost *touching* the horizon. The energy (energy-at-infinity) delivered to the black hole is minimized when the object is released to fall in from this flotation point [7]. One should therefore evaluate  $\mathcal{E}$  at the point  $\ell = b$ .

An assimilation of the charged object results with a change  $\Delta M = \mathcal{E}$  in the black-hole

mass and a change  $\Delta Q = q$  in its charge. The condition for the black hole to preserve its integrity after the assimilation of the body is:

$$q + Q \leq M + \mathcal{E} . \quad (2)$$

Substituting  $\mathcal{E} = \mathcal{E}_0 + \mathcal{E}_{elec} + \mathcal{E}_{self}$  from Eq. (1) one finds a necessary and sufficient condition for removal of the black-hole horizon:

$$(q - \varepsilon)^2 + \frac{2\varepsilon}{M} \left( \mu b - q^2 - \frac{qb^2}{2M} \right) + \frac{q\varepsilon^2}{M} < 0 , \quad (3)$$

where  $r_{\pm} \equiv M \pm \varepsilon$ . The expression on the l.h.s. of Eq. (3) is minimized for  $q = \varepsilon + O(\varepsilon^2/M)$ , yielding

$$2\mu b - q^2 - qb^2/M < 0 , \quad (4)$$

as a necessary and sufficient condition for elimination of the black-hole horizon. This condition (together with the requirement  $b \leq \ell^*$ , the case  $\ell^* < b$  is discussed below) simply implies that the charged object must be smaller than its classical radius. However, any charged body which respects the weak (positive) energy condition (i.e., it does not have a region of negative energy density in it) must be larger than its classical radius. We therefore conclude that the black-hole horizon cannot be removed by an assimilation of such a charged body – cosmic censorship is upheld !

We emphasize that the *quantum* buoyancy due to the Unruh-Wald radiance is a crucial ingredient in this analysis. Without it one could have *slowly* lowered the object down to the horizon (thereby completely redshifting its mass-energy), and it would have been possible to violate cosmic censorship (together with a violation of the GSL).

If the radius of the charged object is larger than  $\ell^*$ , then it must have a minimal energy of  $\mathcal{E}_{min} = \mathcal{E}(\ell^*)$  in order to overcome the potential barrier, and to be captured by the black hole (recall that the effective potential barrier has a maximum located at  $\ell = \ell^*$ ). This is also true for any charged object which is released to fall freely from  $\ell > \ell^*$ , in which case the Unruh-Wald buoyancy can be made arbitrarily negligible (if  $\ell \gg b$ ). Taking cognizance of

Eq. (4) (with  $b$  replaced by  $\ell^*$ ) we find that a necessary and sufficient condition for removal of the black-hole horizon in this case is  $2\mu\ell^* - q^2 - q\ell^{*2}/M < 0$ , or equivalently,

$$\mu^2/q^3 < E, \quad (5)$$

where  $E = Q/r_+^2 = M^{-1} + O(\varepsilon/M^2)$  is the black-hole electric field in the vicinity of its horizon.

The assimilation of a charged object by a charged black hole satisfying condition (5) would *violate* the cosmic censorship conjecture. There is no classical effect that could prevent this. However, Schwinger discharge (vacuum polarization), a purely *quantum* effect sets an upper bound to the black-hole electric field and saves cosmic censorship. Pair-production of the *lightest* charged particles imply a maximal (critical) electric field:  $E \leq E_c \equiv \pi m_e^2/|e|\hbar$ , where  $m_e$  and  $e$  are the rest mass and charge of the electron, respectively. A necessary condition for a violation of the cosmic censorship conjecture within the framework of a quantum theory is the existence of a charged object which satisfies the inequality

$$q^3 E_c/\mu^2 > 1. \quad (6)$$

Obviously, the most dangerous threat to the integrity of the black hole is imposed by the electron, which has the largest charge-to-mass ratio in nature. However, even the electron itself satisfies the relation  $q^3 E_c/\mu^2 = \pi\alpha < 1$  (where  $\alpha = e^2/\hbar \simeq 1/137$  is the fine structure constant), and thus it *cannot* remove the black-hole horizon. Atomic nuclei, the densest composite charged objects in nature satisfy the relation  $q^3 E_c/\mu^2 \lesssim 10^{-7}$  and are therefore absolutely harmless to the black hole. Thus, vacuum polarization (Schwinger discharge of the black hole) insures the integrity of the black hole. Without this *quantum* mechanism one could have removed the black-hole horizon, thereby exposing a naked singularity. It seems that nature has “conspired” to prevent this.

We have shown that two purely quantum effects – Unruh radiation and Schwinger discharge are essential for saving cosmic censorship. Is there any classical effect that we have neglected that could save cosmic censorship? In the analysis presented so far we have assumed that corrections to the metric do not effect the particle’s energy to order  $O(q^2)$ . A

correction of this order would modify condition (3) in such a way that it will be either always satisfied (in which case it would be always possible to violate cosmic censorship regardless of quantum effects) or that it will always be violated (making cosmic censorship viable on a classical level). However, we expect that there is no correction to the particle's energy of order  $O(q^2)$  (except of the self-energy term  $\mathcal{E}_{self}$ , which we have already taken into account). In our analysis we have considered the motion of the particle on the unperturbed metric. In the other extreme case the particle would move on the modified metric with the corrected parameters  $M \rightarrow M + qQ/r_+ + O(q^2/M)$  and  $Q \rightarrow Q + q$ . In this final metric corrections of order  $O(q)$  are canceled out, and the metric is corrected only to the order of  $O(q^2)$ , thereby yielding only a correction of order  $O(q^3)$  to particle's energy.

Although the question of whether cosmic censorship holds remains very far from being settled, we find from this gedanken experiment that the black-hole event horizon may be *classically* unstable while absorbing charged objects. This suggests that the purely classical laws of general relativity do not enforce cosmic censorship. However, *quantum* effects insure the stability of the black-hole event horizon, and thereby restore the validity of the cosmic censorship principle. We thus conclude that the cosmic censor must be cognizant of *quantum gravity*.

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## REFERENCES

- [1] R. Penrose, Riv. Nuovo Cimento **1**, 252 (1969); R. Penrose in *General Relativity, an Einstein Centenary Survey*, eds. S. W. Hawking and W. Israel (Cambridge University Press, 1979).
- [2] R. M. Wald, “Gravitational Collapse and Cosmic Censorship”, e-print gr-qc/9710068; T. P. Singh, “Gravitational Collapse, Black Holes and Naked Singularities”, e-print gr-qc/9805066.
- [3] W. A. Hiscock, Ann. Phys. (N. Y.) **131**, 245 (1981); J. D. Bekenstein and C. Rosenzweig, Phys. Rev. D **50**, 7239 (1994); V. E. Hubeny, Phys. Rev. D **59**, 064013 (1999); T. C. Quinn and R. M. Wald, Phys. Rev. D **60**, 064009 (1999); S. Hod, Phys. Rev. D **60**, 104031 (1999); S. Hod, e-print gr-qc/9908004.
- [4] J. D. Bekenstein, Phys. Rev. D **9**, 3292 (1974).
- [5] R. Wald, Ann. Phys. (N. Y.) **83**, 548 (1974).
- [6] A. I. Zel’nikov and V. P. Frolov, Sov. Phys. -JETP **55**, 191 (1982); D. Lohiya, J. Phys. A: Math. Gen. **15**, 1815 (1982).
- [7] W. G. Unruh and R. M. Wald, Phys. Rev. D **25**, 942 (1982).
- [8] J. D. Bekenstein, Phys. Rev. D **49**, 1912 (1994).
- [9] J. Schwinger, Phys. Rev. **82**, 664 (1951); M. A. Markov and V. P. Frolov, Teor. Mat. Fiz. **3**, 3 (1970); G. W. Gibbons, Comm. Math. Phys. **44**, 245 (1975); T. Damour and R. Ruffini, Phys. Rev. Lett. **35**, 463 (1975).